CONVERGENCE TO EQUILIBRIUM FOR SOME DISCRETIZED GRADIENT OR GRADIENT-LIKE SYSTEMS

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A celebrated result of Łojasiewicz asserts that if $F : \mathbb{R}^d \to \mathbb{R}$ is real analytic, then any bounded solution $U \in C^1([0, +\infty), \mathbb{R}^d)$ of the gradient flow

$$U'(t) = -\nabla F(u(t)), \quad t \geq 0, \tag{1}$$

converges to equilibrium as t tends to infinity. A similar result holds for the second order gradient-like flow

$$\epsilon U''(t) + U'(t) = -\nabla F(u(t)), \quad t \geq 0, \tag{2}$$

where ϵ is a positive constant. Here we are concerned with the following question: is it possible to adapt these results for some stable time discretizations of (1) or (2) ? In particular, what happens for the backward Euler scheme ? Some applications to PDEs such as the Allen-Cahn equation, the Cahn-Hilliard equation, and other closely related equations will also be considered.