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Brennan's conjecture for weighted composition operators

Brennan's conjecture concerns integrability of the derivative of a conformal map τ of the unit disk $\mathbf{D}$. The conjecture is that, for all such τ,

$$\int_{\mathbf{D}} (1/|\tau'|)^p dA < \infty$$

holds for $-2/3 < p < 2$. This is known for $-2/3 < p \leq 1.421$.
We show Brennan's conjecture is equivalent to a statement about weighted composition operators. Let τ be as above and let φ be an analytic self-map of $\mathbf{D}$. Define, for f analytic on $\mathbf{D}$,

$$(A_{\varphi,p} f)(z) = \left(\frac{\tau'(\varphi(z))}{\tau'(z)} \right)^p f(\varphi(z)).$$

There are always choices of φ that make $A_{\varphi,p}$ *bounded* on the Bergman space $L^2_a(\mathbb{D})$. We are interested in the set of p for which there is a choice of φ (depending on τ) that makes $A_{\varphi,p}$ *compact* on $L^2_a(\mathbb{D})$. We show this happens if and only if $(1/\tau')^p \in L^2_a(\mathbb{D})$. Thus Brennan's conjecture is equivalent to such a choice of φ existing for the range $-1/3 < p < 1$, and this is known for $-1/3 < p \leq .7105$.