COMPLEX APPROXIMATION AND UNIVERSALITY

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Abstracts

Universal functions on $H^\infty(B^n)$

Richard Aron

We will report on a fairly recent paper with Pamela Gorkin, whose principal result is a theorem about universal functions on $H^\infty(B^n)$, where B^n is the ℓ_2-ball in $\mathbb{C}^n$. P. S. Chee showed that there is a sequence (L_k) of automorphisms of B^n to which one can associate a universal function $f \in H^\infty(B^n)$, $\|f\| = 1$. That is, the set $\{f \circ L_k \mid k \in \mathbb{N}\}$ is dense in $H^\infty(B^n)$ when this space is endowed with the compact-open topology. Here, each L_k corresponds to a point $z_k \in B^n$.
The topic addressed here is the *size* of the set of such universal functions f.
Theorem: There is a sequence $(z_k) \subset B^n$ for which one can find an infinite dimensional closed subspace $V \subset H^\infty(B^n)$ with the following property: Every $f \in V$, $\|f\| = 1$, is universal with respect to the sequence (L_k) (joint work with Pamela Gorkin)

Hypercyclic operators failing the Hypercyclicity Criterion

Frédéric Bayart

It was a long-standing open problem to know whether, for every hypercyclic operator, its direct sum with itself remains hypercyclic. This problem has been recently solved in the negative by De La Rosa and Read. We show how to construct this kind of operators on a Hilbert space and how to obtain various qualitative properties of them (joint work with E. Matheron).

Which maps preserve universal functions?

George Costakis

We examine whether there exist maps, between spaces of holomorphic functions, which preserve certain notions of universality. For instance, we show that if f is a universal Taylor series in the unit disk in the sense of Luh and Chui and Parnes and p is a polynomial having all its roots on the closed unit disk then the function pf is also a universal Taylor series in the unit disk. We prove similar results for the class of universal functions with respect to derivatives and several open problems will be discussed.

Asymptotic maximum principles for subharmonic functions

Stephen J. Gardiner

A classical result of Valiron says that an unbounded holomorphic function on the unit disc, which is bounded on a spiral that accumulates at all boundary points, must tend to ∞ along some boundary path. Barth and Rippon have recently obtained more general results of this nature for unbounded holomorphic functions which are bounded on a set that accumulates nontangentially at a large set of boundary points. We will explain how some of their results follow from very general maximum principles for subharmonic functions that are based on asymptotic behaviour. The sharpness of these maximum principles can be verified using approximation arguments.

A universal solution for a Quasi-linear Parabolic Equation occuring in aerodynamics

Paul M. Gauthier

We shall present results on universal functions as well as universal (formal) series for solutions to a quasi-linear parabolic equation arising in aerodynamics, Burgers' equation. Burgers' equation is one of the simplest examples of a non-linear partial differential equation and also perhaps the simplest equation describing waves under the influence of diffusion. The crucial role in our investigation is due to the so-called Cole-Hopf transformation. (Joint work with N. Tarkhanov)

Lacunary summability and analytic continuation of power series

Tatevik L. Gharibyan

In this talk summability methods of weighted mean type are going to be discussed which are generated by lacunary sequences. A result on the summability of the geometric series is proved which has far-reaching consequences with respect to general analytic continuation of power series into their Mittag-Leffler-star, to overconvergence as well as to the universal behavior of trigonometric series in the sense of Menšov.

Construction versus Baire category in universality

Karl-Goswin Grosse-Erdmann

The following question is explicitly or implicitly posed in several papers on universality: In existence proofs of universal functions, is it preferable to construct a specific example or should one use the Baire category theorem? We shall argue that the two approaches are largely equivalent. Our discussion leads us, among other things, to the solution of two problems of Kariofillis, Nestoridis, Aldred and Armitage, to a re-evaluation of a theorem of Herzog on restricted universality, and to improvements of some recent results on universal functions.

Approximating inner functions

Geir Arne Hjelle

It follows from a well known theorem of Otto Frostman that inner functions can be uniformly approximated by Blaschke products. Whether the same can be done with interpolating Blaschke products is an open question, that has received interest for more than 25 years.
I will highlight some of the work done on this problem, and in particular explain a construction where we prove that the modulus of inner functions can be uniformly approximated by the modulus of interpolating Blaschke products.

A new method for non-supercyclicity

Fernando León-Saavedra

A bounded linear operator T defined on a Banach space X is said to be supercyclic if there exists $x \in X$ such that $\{\lambda T^n x : n \in \mathbb{N}, \lambda \in \mathbb{C}\}$ is dense in X, and it is called weakly supercyclic the the above set is weakly dense in X. The interest in the study of different types of operator orbits arises from the invariant subspace problem. To prove non-supercyclicity usually is very complicated. Our results assert that under certain conditions supercyclicity is equivalent to positive supercyclicity: Namely, we can multiply $T^n x$ only by positive real numbers. As a consequence we obtain a new approach to the invariant subspace problem in the positive direction. Moreover, we provide a new technique to provide non-supercyclicity, even non weakly-supercyclicity. This method applies to a large class in the commutant of the classical Volterra operator is not weakly supercyclic, the infinite Cesàro operator, etc. The size of the commutant of T will be important in the discussion.

Densities connected with overconvergence

Wolfgang Luh

It is well known that universal Taylor series have a strong connection with overconvergence. While by Ostrowski's classical results there exist interdependencies with the occurence of gaps in the sequence of coefficients and the phenomenon of overconvergence.
We here investigate the structure of corresponding gaps and especially we study density properties for gap intervals as well as for "non-gaps" intervals. (Joint work with T. Gharibyan)

Universal functions for composition operators

Raymond Mortini

Let Ω be a planar domain and let $H(\Omega)$ be the Fréchet space of all holomorphic functions on Ω. Let X denote either $H(\Omega)$ or the unit ball $\mathcal{B} = \{f \in H(\Omega) : \sup_{z\in\Omega} |f(z)| \leq 1\}$ of $H^\infty(\Omega)$. A function $f \in X$ is said to be X-universal for a sequence, (ϕ_n), of selfmaps of Ω if $\{f \circ \phi_n : n \in \mathbb{N}\}$ is (locally uniformly) dense in X. Whereas the case of ϕ_n being an automorphism has been succesfully dealt with by many authors, we will study here the general case. It will be shown that for every domain $\Omega \subseteq \mathbb{C}$ for which $H^\infty(\Omega)$ is dense in $H(\Omega)$ there exists a sequence (ϕ_n) such that the family (C_{ϕ_n}) of composition operators admits $H(\Omega)$-universal functions. Moreover, if Ω is finitely connected, but not simply connected, then such a sequence of selfmaps cannot be eventually injective. On the other hand, if Ω is a domain of infinite connectivity, then a sequence of eventually injective selfmaps of Ω admits $H(\Omega)$-universal functions if and only for every Ω-convex compact subset K of Ω and every $n \in \mathbb{N}$ there is some $n \geq N$ such that $\phi_n(K)$ is Ω-convex and $\phi_n(K) \cap K = \emptyset$. The case of simply connected domains is considered, too. The problem of characterizing $\mathcal{B}$-universality for selfmappings of a non-simply connected domain is still open.

Uniform approximation by interpolating Blaschke products

Raymond Mortini

Let $\mathfrak{I}$ be the class of all inner functions that can be uniformly approximated on $\mathbb{D}$ by interpolating Blaschke products. It is well known that any infinite Blaschke product whose zeros lie in a cone belongs to $\mathfrak{I}$. It will be shown that any inner function u for which there exists a level set $\{|u| < \eta\}$ that can be controlled in a certain way by the zero *set* of u belongs to $\mathfrak{I}$. In particular, we will notice that $\mathfrak{I}$ contains the set of inner functions satisfying the weak embedding property; a set that appeared in recent work of Gorkin, Nikolski and the speaker on H^∞-quotient algebras.

From Polynomial Approximation to Universal Taylor Series and Back Again

Jürgen Müller

Let Ω be a simply connected domain in the complex plane with $0 \in \Omega$. It is well known that results on polynomial approximation may be used to prove the existence of functions in $H(\Omega)$ such that the sequence (s_n) of the partial sums of the Taylor expansion around 0 has universal approximation properties outside Ω. On the other hand, such universal approximation of (s_n) implies that certain subsequences $(s_{n_k})_k$ approximate f in Ω. The talk focusses on this interplay.

Universal series in $\bigcap\limits_{p>1} \ell^p$

V. Nestoridis

Exploiting the recently developed “Abstract Theory of universal series” we give a sufficient condition so that there exists a universal series in $\bigcap\limits_{p>1} \ell^p$. In particular we have approximation by translates of the Riemann zeta function in $\mathbb{C}$ or by translates of a fundamental solution of suitable elliptic operators with constant coefficients in $\mathbb{R}^n$, or by translates of approximate identities in $\mathbb{R}^d$, as for examples, by normal distributions. An application yields universal trigonometric series in $\mathbb{R}^d$ (non-periodic case) with frequencies with finite accumulation points. Improvements of these results are obtained by using universal Dirichlet series in one or several variables, where the only accumulation point of the frequencies is ∞.

An improvement of the universality of the geometric series

V. Nestoridis

Recently A. Mouze obtained an improvement of a result of Bernal–Gonzalez, Calderon-Moreno and W. Luh concerning the universality of the geometric series. Mouze proved the following: Let $S = \sum_{n=0}^{\infty} c_n z^n$ be a formal power series with $c_n \neq 0$ for all $n \in \mathbb{N}$. Then there exists a matrix $A = [a_{n,v}]_{n,v \geq 0}$, satisfying some properties, such that the sequence of the A-transforms of S has universal properties in $\{z : |z| \geq 1\} - \{1\}$. The set of universal series with respect to this matrix A is D_δ and dense in natural spaces of series. Similar results are obtained replacing power series by Dirichlet series.

Close universal approximants of the Riemann zeta-function

Markus Niess

The Riemann zeta-function $\zeta(z)$ has the following well-known properties:

(1) It is holomorphic in the complex plane except for a single pole at $z = 1$ with residue 1.
(2) The symmetry relation $\zeta(z) = \overline{\zeta(\bar{z})}$ holds for $z \neq 1$.
(3) The functional equation

$$\zeta(z)\Gamma(z/2)\pi^{-z/2} = \zeta(1-z)\Gamma((1-z)/2)\pi^{-(1-z)/2}$$

holds.
(4) It has a universal property due to Voronin (1975).

We show that arbitrarily close approximations of the Riemann zeta-function which satisfy (1)-(3) may have a different universal property. Consequently, these approximations do not satisfy the Riemann hypothesis. Moreover, we investigate the set of all "‘Birkhoff-universal"’ functions satisfying (1)-(3).

Universal Series and Fundamental Solutions of the Laplace Equation

Innocent Tamptse

Let ϕ be the standard fundamental solution of the Laplace operator on $\mathbf{R}^N, (N \geq 2)$. We prove the existence of universal series of the form

$$\sum_{k=0}^{\infty} c_k \phi(x - a_k) \tag{1}$$

$$\sum_{k=0}^{\infty} \sum_{|\alpha|=k} c_\alpha D^\alpha \phi(x - a) \tag{2}$$

in the space of functions that are harmonic in the neighborhood of a fixed compact set $K \subset \mathbf{R}^N$ with connected complement, or the space of functions that are harmonic on an open set $\Omega \subset \mathbf{R}^N$ that have an exhaustion by compact sets with connected complements. We also prove the existence of a serie of the form (2) which is convergent in $\mathbf{R}^N \setminus B(0, r)$ and is universally overconvergent in $B(a, r)\setminus\{a\}$. $a, a_k, k \in \mathbf{N}$ are fixed points lying outside the domain of definition of the harmonic functions. We also give some conditions for series in the form

$$\sum_{k=0}^{\infty} \sum_{|\alpha|=k} c_\alpha u_\alpha \tag{3}$$

to be universal in a metrizable topological linear space X.

Universal Faber series

Vagia Vlachou

For a certain type of doubly connected domains, we prove that there exist functions, holomorphic on such a domain, such that for any choice of compact set with connected complement in the domain, the corresponding Faber series is universal.

More specifically, if $K \subset \mathbb{C}$ is a compact and connected set with connected complement, containing more than one points and $\Omega = \mathbb{C} \setminus K$, then the class $\bigcap_{\Gamma \in Y} U(\Omega, \Gamma)$ is residual in $H(\Omega)$, where

$$Y = \{\Gamma \subset \Omega : \Gamma \text{ is compact and connected set, containing more than one points and } \mathbb{C} \setminus \Gamma \text{ is connected.}\}$$

Remark: By $U(\Omega, \Gamma)$, we denote the class of universal Faber series in respect to Γ, that is the corresponding Faber expansion of a function in the class, realizes approximations outside Ω.